Zoom Class 1/5/2020

Asymptotes of Polar Curves

General equation of asymptotes are

$$\boxed{p = r\cos(\theta - \alpha)}$$

and $Lt\left[-\frac{d\theta}{du}\right] = r\cos\left[\theta - \theta_1 + \frac{\pi}{2}\right]$

$\Rightarrow \boxed{Lt\left[-\frac{d\theta}{du}\right] = r\sin(\theta_1 - \theta)}$ where $\theta \to \theta_1$, $r \to \infty, u \to 0$

Working Rule

(i) Put $r = \frac{1}{u}$ in the given equation and find the limit of θ, when $u \to 0$

(ii) let $\theta = \theta_1$ be any one limit among the various possible limits of θ

(iii) After this, we find the limit of $\left[-\frac{d\theta}{du}\right]$ when $u \to 0$ and $\theta \to \theta_1$

If its limit is p. Then $p = r\sin(\theta_1 - \theta)$ will be the equation of asymptote corresponding to θ_1

Ques Find asymptotes of the curve $r\theta = a$

Soln Here $\theta = \frac{a}{r} = au \Rightarrow \theta = au$ —— (1)

$\therefore$ when $u \to 0$, then $\theta \to 0 \Rightarrow \theta_1 = 0$

Also (1) $\Rightarrow \frac{d\theta}{du} = a$

$\therefore Lt\left(-\frac{d\theta}{du}\right) = r\sin(\theta_1 - \theta) \Rightarrow -a = r\sin(0-\theta) = -r\sin\theta$

$\Rightarrow a = r\sin\theta$. Ans

Ques 4 Find the asymptotes of the curve $r = \frac{2a}{1-2\cos\theta}$.

Soln. Here $u = \frac{1}{r} = \frac{1-2\cos\theta}{2a}$ —— (1)

$\therefore$ while $u \to 0 \Rightarrow 1-2\cos\theta \to 0 \Rightarrow \cos\theta \to 1/2$

$\Rightarrow \theta = \pm \pi/3$

$\Rightarrow \theta_1 = \pm \pi/3$

Now $\frac{du}{d\theta} = \frac{1}{2a}(0+2\sin\theta) = \frac{1}{a}\sin\theta$

$\Rightarrow \frac{d\theta}{du} = \frac{a}{\sin\theta}$

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$\Rightarrow \underset{\theta \to \frac{\pi}{3}}{Lt}\left(-\frac{d\theta}{du}\right) = -\underset{\theta \to \frac{\pi}{3}}{Lt} \frac{a}{\sin\theta} = \frac{-a}{\sin\pi/3}$

$= \frac{-a}{\sqrt{3}/2} = \frac{-2a}{\sqrt{3}}$

and $\underset{\theta \to -\frac{\pi}{3}}{Lt}\left(-\frac{d\theta}{du}\right) = -\underset{\theta \to -\frac{\pi}{3}}{Lt} \frac{a}{\sin\theta} = \frac{-a}{\sin(-\frac{\pi}{3})}$

$= \frac{\cancel{-}a}{\cancel{-}\sin\pi/3} = \frac{a}{\sqrt{3}/2} = \frac{2a}{\sqrt{3}}$.

Hence the required asymptotes are

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$\frac{-2a}{\sqrt{3}} = r\sin\left(\frac{\pi}{3} - \theta\right)$

$\Rightarrow \frac{-2a}{\sqrt{3}} = r\left(\sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta\right)$

$$\Rightarrow \frac{-2a}{\sqrt{3}} = r\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$$

$$= \frac{r}{2}\left(\sqrt{3}\cos\theta - \sin\theta\right)$$

$$\Rightarrow -4a = r\left(3\cos\theta - \sqrt{3}\sin\theta\right) \text{ --- } (2)$$

And $\frac{2a}{\sqrt{3}} = r\sin\left(-\frac{\pi}{3} - \theta\right)$

$$\Rightarrow \frac{2a}{\sqrt{3}} = -r\left[\sin\frac{\pi}{3}\cdot\cos\theta + \cos\frac{\pi}{3}\cdot\sin\theta\right]$$

$$\Rightarrow \frac{2a}{\sqrt{3}} = -r\left[\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right]$$

$$\Rightarrow \frac{2a}{\sqrt{3}} = -\frac{r}{2}\left[\sqrt{3}\cos\theta + \sin\theta\right]$$

$$\Rightarrow -4a = r(3\cos\theta + \sqrt{3}\sin\theta) \text{ --- } (3)$$

Ans